Phase Noise Minimization of Microwave Oscillators by Optimal Design

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Abstract—**A novel time domain phase noise analysis and minimization method (TDPNAM) is presented and applied to the design of a 15 GHz microstrip line oscillator. The new method determines the design, e.g., the linear network of oscillators with a minimized phase noise by solving an appropriate optimal control problem numerically. Starting from a design with standard CAD-tools the measured single-sideband phase noise is reduced by 10 dB over the whole offset frequency range to values as low as -100 dBc/Hz at an offset frequency of 100 kHz.**

I. Introduction

Up to now in oscillator design the minimization of the phase noise has been done using empirical rules [1, 2, 3, 4], e.g., increasing the loaded quality factor of the resonator, matching the noise impedance, or achieving a balanced characteristic of the nonlinearities. The combination of the harmonic balance method with optimization methods to minimize the phase noise has been reported [5], but not demonstrated with measured data. We present a novel time domain method to optimize the phase noise of oscillators over the whole offset frequency range, show its application to the design of oscillators and compare the calculations with experimental results of the fabricated oscillators. For the numerical calculations the differential equations describing the oscillator circuit are discretized and an approximation of the function to be minimized which calculates the phase noise is used. The advantage of the method is that it can be performed without the repeated computation of the whole algorithm for each set of optimization parameters like it has to be done using harmonic balance techniques. As the method to optimize the phase noise of oscillators is a time domain technique it can be implemented in time domain simulation programs, as e.g., SPICE.

II. Theoretical Approach

The method to minimize the phase noise of oscillators is based on the description of the signal and noise behavior by the Langevin equations

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \boldsymbol{\xi}(t), y(t)) = \\ &= \mathbf{f}(\mathbf{x}(t)) + \mathbf{G}(\mathbf{x}(t))\boldsymbol{\xi}(t) + \mathbf{g}(\mathbf{x}(t))y(t) + \\ &\quad + \mathcal{O}(\boldsymbol{\xi}^2, y^2, \boldsymbol{\xi}y), \qquad (1)\\ \mathbf{G}(\mathbf{x}(t)) &= \left.\frac{\partial \mathbf{f}(\mathbf{x}(t), \boldsymbol{\xi}(t), y(t))}{\partial \boldsymbol{\xi}}\right|_{\boldsymbol{\xi}=0, y=0}, \\ \mathbf{g}(\mathbf{x}(t)) &= \left.\frac{\partial \mathbf{f}(\mathbf{x}(t), \boldsymbol{\xi}(t), y(t))}{\partial y}\right|_{\boldsymbol{\xi}=0, y=0},\end{aligned}$$

where $\mathbf{x}$ are the state variables of the circuit, $\boldsymbol{\xi}$ are the white noise sources and y is a nonlinear $f^{-\alpha}$ noise source. Due to the assumption of small noise sources compared to the signal amplitudes the terms of order $\mathcal{O}(\boldsymbol{\xi}^2, y^2, \boldsymbol{\xi}y)$ are neglected.

The single-sideband phase noise $L(f_m)$ of oscillators is calculated by solving eq. (1) using a perturbation approach [6]

$$\begin{aligned}L(f_m) &= \frac{\Delta f_{3dB}}{\pi f_m^2} + \omega_0^2 |g_{1,0}|^2 \frac{k}{|2\pi f_m|^{2+\alpha}} \qquad (2)\\ \text{with } \Delta f_{3dB} &= \frac{1}{4\pi}\omega_0^2 \frac{1}{T_0}\int_0^{T_0} \mathbf{v}(\mathbf{x}(t))^T \mathbf{G}(\mathbf{x}(t)) \cdot \\ &\quad \cdot \boldsymbol{\Gamma}(\mathbf{x}(t)) \mathbf{G}(\mathbf{x}(t))^T \mathbf{v}(\mathbf{x}(t)) dt \\ \text{and } g_{1,0} &= \frac{1}{T_0}\int_0^{T_0} \mathbf{v}(\mathbf{x}(t))^T \mathbf{g}(\mathbf{x}(t)) dt.\end{aligned}$$

The first term on the right hand side of eq. (2) describes the phase noise caused by the unmodulated and modulated white noise sources. The vector $\mathbf{v}(\mathbf{x}(t))$ is the left-sided eigenvector corresponding to the eigenvalue 1

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of the fundamental matrix $\boldsymbol{\Psi}(T_0, 0)$ [7]. The matrix $\boldsymbol{\Gamma}$ consists of the correlation functions of the white noise sources. The second term on the right hand side of eq. (2) describes the phase noise caused by the baseband noise. The modulation of the nonlinear $f^{-\alpha}$ noise source and the upconversion of the baseband noise caused by the nonlinearities in the circuit are taken into account. The factor k is determined by baseband noise measurements.

The optimization of the phase noise can be formulated with time domain techniques as a so-called optimal control problem [8]. The aim of optimal control problems is to determine a control vector function $\mathbf{u}(t)$ that minimizes a functional

$$\int_0^T h(\mathbf{x}(t))\,\mathrm{dt} \rightarrow \min, \qquad (3)$$

where the state variables $\mathbf{x}(t)$ and $\mathbf{v}(\mathbf{x}(t))$ have to satisfy the differential equations

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \qquad (4) \\ \dot{\mathbf{v}}(\mathbf{x}(t)) &= -\left(\frac{\partial \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))}{\partial \mathbf{x}(t)}\right)^T \mathbf{v}(\mathbf{x}(t)) \end{aligned}$$

and the boundary conditions

$$\mathbf{r}(\mathbf{x}(0), \mathbf{x}(T)) = \mathbf{0}\;. \qquad (5)$$

In our case $\mathbf{u}(t)$ is a constant vector $\mathbf{p}$ of the circuit parameters, e.g. capacitors, inductors or resistors of the circuit. The boundary conditions (5) are periodic boundary conditions,

$$\mathbf{r}(\mathbf{x}(0), \mathbf{x}(T)) = \mathbf{x}(0) - \mathbf{x}(T) = \mathbf{0}\;, \qquad (6)$$

with the period time T. Due to eq. (2) the phase noise of an oscillator can be calculated as an integral of a function $h(\mathbf{x}(t))$ depending on the state variables and the correlation functions of the noise sources.

$$L(f_m, \mathbf{p}) = \int_0^T h(\mathbf{x}(t), \mathbf{p})\,\mathrm{dt} \qquad (7)$$

Optimal control problems appear in many technical applications [8]. Therefore much interest has been given to the development of efficient numerical solution methods. Here, we used a so-called direct collocation method that has shown to be robust and reliable in solving such complicated problems [9]. The method is based on an approximation of the state variables $\mathbf{x}(t)$ and $\mathbf{v}(t)$ by piecewise cubic spline functions $\tilde{\mathbf{x}}(t)$ and $\tilde{\mathbf{v}}(t)$ with the coefficients $\mathbf{c}$. The parameters $\mathbf{p}$ of the circuit and the coefficients $\mathbf{c}$ have to be determined in order to minimize the phase noise

$$L(f_m, \mathbf{p}) = \int_0^T h(\mathbf{c}, \mathbf{p})\,\mathrm{dt} \qquad (8)$$

subject to the differential equations

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t_i) &= \mathbf{f}(\tilde{\mathbf{x}}(t_i), \mathbf{p})\,, \qquad i = 1, \ldots, N \qquad (9) \\ \dot{\tilde{\mathbf{v}}}(t_i) &= -\left(\frac{\partial \mathbf{f}(\tilde{\mathbf{x}}(t_i), \mathbf{p})}{\partial \tilde{\mathbf{x}}(t_i)}\right)^T \tilde{\mathbf{v}}(t_i) \end{aligned}$$

and the periodic boundary conditions

$$\tilde{\mathbf{x}}(0) = \tilde{\mathbf{x}}(T)\;; \quad \tilde{\mathbf{v}}(0) = \tilde{\mathbf{v}}(T)\;. \qquad (10)$$

The differential equations (9) are only fulfilled at the so-called collocation points: the discretization points t_i, $i = 1, \ldots, N$, and at the centers $(t_i + t_{i+1})/2$ between. It should be noted that (9) are nonlinear equations in the unknown coefficients $\mathbf{c}$ and the parameters $\mathbf{p}$.

The resulting nonlinear optimization problem is solved by an iterative algorithm based on Sequential Quadratic Programming (SQP). SQP methods have found wide-spread usage for the solution of nonlinearly constrained optimization problems [10]. In each iteration a quadratic problem consisting of a quadratic approximation of the function to be minimized and linear constraints that result from linearization of equation (9) and boundary conditions (10) has to be solved to calculate the next iteration step. The direct collocation method has shown to be easy to use, because the knowledge of optimal control theory is not required, and robust, because not much information on the solution of the steady state is a priori needed.

III. Experimental Results

The TDPNAM method is applied to a planar integrated [11] free running microwave oscillator at 15 GHz. As the active element a GaAs MESFET is used and the resonator consists only of microstrip lines at the gate and source terminals of the MESFET. The nonlinearities are described with a modified SPICE model [12, 13]. The equivalent circuit with the nonlinearities of the FET is depicted in Fig. 1. The microstrip lines are modelled in the vicinity of the carrier frequency with lumped element equivalent circuits, which are depicted in Fig. 2. The layout of the microstrip lines is shown in Fig. 3, where the microstrip line at the gate terminal of the transistor is on the left hand side and the microstrip line at the drain terminal is located at the right hand side of Fig. 3.

White noise sources are the thermal noise sources of the losses and the shot noise sources of the internal diodes. A bias dependent $f^{-\alpha}$ current noise source parallel to the nonlinear current source I_{DS} is modelled due to baseband noise measurements. The calculation of the single-sideband phase noise of this oscillator with time-domain and frequency-domain techniques is already

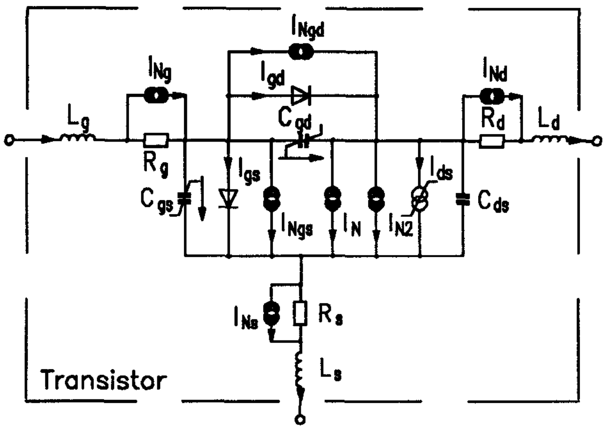

Figure 1: Equivalent circuit of the FET.

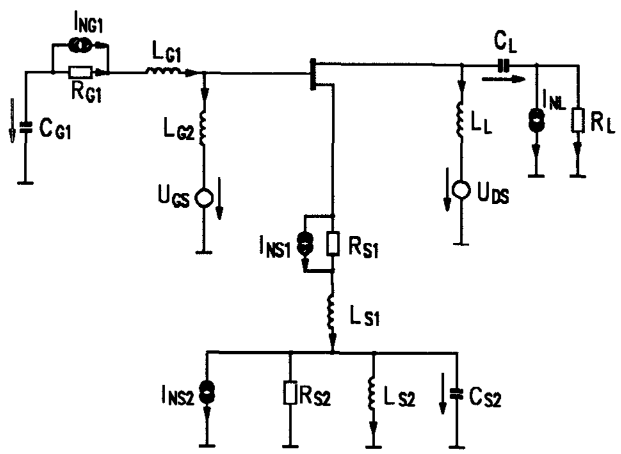

Figure 2: Equivalent lumped element circuit of the linear oscillator subcircuit.

published in [14]. The computing time to calculate the single-sideband phase noise on a HP 700 workstation is about fifteen seconds, mainly to calculate the steady state.

For the minimization of the phase noise the microstrip lines at the gate and source terminal are optimized. They are modelled with lumped element equivalent circuits consisting of a series resonant circuit at the gate and a parallel and series resonant circuit at the source. With the equivalent circuit of the transistor 20 nonlinear differential equations are obtained by applying the described optimization method. Five parameters of the lumped element equivalent circuit, L_{G1}, C_{G1}, L_{S1}, L_{S2} and C_{S2}, are chosen to be optimized. As the quality factors of microstrip lines with different layouts are almost constant, the impedances R_{G1}, R_{S1} and R_{S2} are chosen respectively. The function $L(f_m)$ in eq. (8) has to be minimized where the phase noise near the

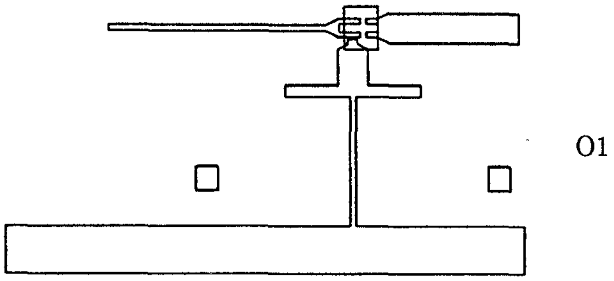

Figure 3: Layout of the oscillator to be optimized O1 with a scaling factor of about 10:1.

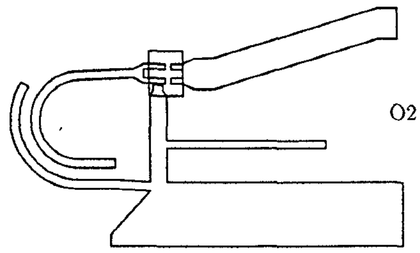

Figure 4: Layout of the optimized oscillator O2 with a scaling factor of about 10:1.

carrier caused by the baseband noise and the phase noise at high offset frequencies which is caused by white noise sources are taken into account. The optimization needs about 100 CPU minutes on a HP 700 workstation. The values of the optimization parameters before and after the optimization are shown in Table 1. The

	before		after	
	optimization			
L_{G1}	1.127	nH	3.30	nH
C_{G1}	0.170	pF	0.037	pF
L_{S1}	251.8	pH	181.6	pH
L_{S2}	1.207	nH	0.40	nH
C_{S2}	253.5	fF	500.0	fF

Table 1: Values of the optimization parameters before and after the optimization.

optimized lumped element equivalent circuits are realized with distributed microstrip circuits again, but different layouts have to be used. By this way not only the lengths and widths of the microstrip lines are optimized but also a different and better layout with a different topology can be found. The layouts of both oscillators are depicted in Fig. 3 and 4. The optimized oscillator has an output power of 13.3 dBm compared with the oscillator to be optimized of 12.7 dBm.

The simulated and measured single-sideband phase noise of both oscillators is depicted in Fig. 5. A significant reduction of over 10 dB of the measured phase noise is achieved and a single sideband phase noise of -100 dBc/Hz at a frequency deviation of 100 kHz is obtained, which is a very low phase noise compared with the literature [15]. The measurement of the optimized phase noise was only possible up to 2 MHz because of a very low IF-frequency of the phase noise measurement system HP 3048. The measured peaks between 400 kHz and 8 MHz are due to external interferences. At the oscillatory frequency the resonator at the gate and source terminal of the transistor can be simulated by a basic series resonant circuit consisting of L_0, C_0 and R_0. The quality factor $Q = \omega_0 L_0 / R_0$ of the manufactured resonators change only from 144 to 177. The impedances of the microstrip lines between the terminals of the transistor and ground change only from about $j40\ \Omega$ and $-j47\ \Omega$ for the gate and source terminal to about $j27\ \Omega$ and $-j37\ \Omega$. But the significant difference between the two oscillators is the increase of the ratio L_0/C_0 from $29 \cdot 10^3\ \Omega^2$ to $884 \cdot 10^3\ \Omega^2$. It can be shown for a basic van der Pol-oscillator with a series resonant circuit that the single-sideband phase noise is proportional $1/(L_0/C_0)$. By using the described optimization method the highest ratio of L_0/C_0 for a complicated resonator structure can be found which still provides a stable oscillation.

Conclusion

The presented TDPNAM method is a general method for the optimum design of oscillators with minimized phase noise. The minimum phase noise design is obtained systematically by numerical optimization. The method has been applied to microwave oscillators and more than a 10 dB reduction of the phase noise over the complete measurable offset frequency range compared with a conventional design was achieved for both the simulation and the experiment. This significant reduction is possible only by an optimized layout of microstrip lines while the quality factor of the resonator remains almost constant.

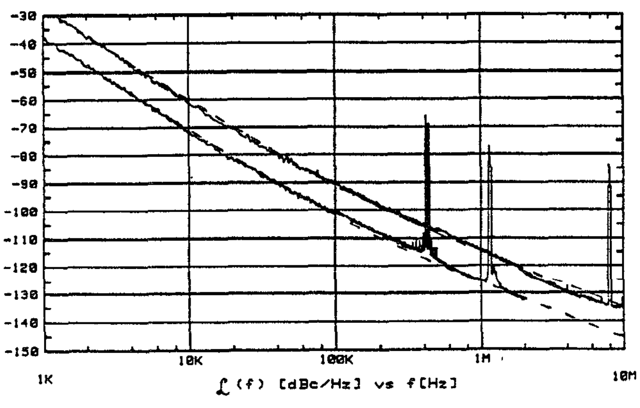

Figure 5: Measured and simulated single sideband phase noise of the oscillator to be optimized O1 (– – –) and the optimized oscillator O2 (–·–·–).

Acknowledgement

The authors gratefully acknowledge the support by the DFG and FORTWIHR.

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